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CLASS XII (2024-25)
APPLIED MATHEMATICS (241)
POST MID-TERM EXAM
SET-A

Max Marks:80 **Time:3 HRS**

General Instructions: Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.
(ix) Use of calculators is not allowed.

SECTION A
(Multiple Choice Questions) Each question carries 1 mark.

1. Sampling which provides for a known non zero equal chance of selection is:
   a) Systematic sampling c) Convenience sampling
   b) Quota sampling d) Purposive sampling
2. If the supply function for a commodity is $p = \sqrt{9 + x}$ and the market price $p_0 = 4$, then producer's surplus is:
   a) 3 b) $\frac{10}{3}$ c) 10 d) 15
3. If x is the least non-negative integer satisfying $218 \equiv x(mod\ 7)$, then $(x^2 + 1)$ is equal to
   a) 1 b) 2 c) 5 d) 50
4. If A is a square matrix of order 3 and $|A| = 2$, then the value of $|-AA'|$ is
   a) 4 b) 2 c) -2 d) -4
5. If A and B are symmetric matrices of same order, then $C = AB' - BA'$ is
   a) C b) $-C$ c) O d) I
6. If $f(x) = \log x$, then derivative of $f(\log x) w.r.t.\ x$ is
   a) $\frac{\log x}{x}$ b) $\frac{x}{\log x}$ c) $x \log x$ d) $\frac{1}{x \log x}$

7. The equation of the tangent to the curve $y = e^{2x}$ at $(0,1)$ is

   a) $y + 1 = 2x$ b) $1 - y = 2x$ c) $y - 1 = 2x$ d) $y = x$

8. The minimum value of $x^2 + \frac{250}{x}$ is

   a) 75 b) 55 c) 50 d) 25

9. If the function $f(x) = x^2 - kx + 5$ is increasing $[2,4]$, then

   a) $k \in (2, \infty)$ b) $k \in (-\infty, 2)$ c) $k \in (4, \infty)$ d) $k \in (-\infty, 4)$

10. The solution of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(0) = 2$ is

    a) $y + 3x = 2$ b) $y - 3x = 2$ c) $y + 3x + 2 = 0$ d) $y = 3x$

11. The sum of degree and order of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\left(\frac{d^3y}{dx^3}\right)$ is

    a) 4 b) $\frac{5}{3}$ c) 5 d) 6

12. For a random variable X, $E(X) = 3$ and $E(X^2) = 11$. Then variance of X is

    a) 8 b) 5 c) 2 d) 1

13. A sample of 50 bulbs is taken at random. Out of 50 we found 15 bulbs are of Bajaj, 17 are of Surya and 18 are of Crompton. What is the point estimate of population proportion of Surya?

    a) 0.3 b) 0.34 c) 0.36 d) 0.4

14. If $\alpha =$ probability of Type-I error, then $1 - \alpha$ is

    a) Probability of rejecting H_0 when H_0 is true.
    b) Probability of not rejecting H_0 when H_0 is true.
    c) Probability of not rejecting H_0 when H_a is true.
    d) Probability of rejecting H_0 when H_a is true.

15. A factory production is delayed for three weeks due to breakdown of a machine and unavailability of spare parts. Under which trend oscillation does this situation fall?

    a) Seasonal b) Cyclical c) Secular d) Irregular

16. In an LPP if the objective function $Z = ax + by$ has same maximum value of two corner points of the feasible region, then the number of points at which maximum value of Z occurs is:

    a) 0 b) 2 c) Finite d) Infinite

17. An investment of $Rs.5000$ in securities gave Rs.1250 as income. Then the return on investment will be:

    a) 1:4 b) 4:1 c) 5:1 d) 1:5

18. I wish to receive Rs.3000 at the beginning of every month forever, how much should I invest if the rate of interest is 6% per annum compounded monthly?

    a) Rs.600000 b) Rs.597000 c) Rs.603000 d) Rs.650000

ASSERTION-REASON BASED QUESTION:

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false. (d) A is false but R is true.

19. **Assertion (A):** Minor of an element of a determinant of order n $(n \geq 2)$ is a determinant of order n.
Reason (R): If A is an invertible matrix of order 2, then $\det(A^{-1}) = \frac{1}{|A|}$.

20. **Assertion (A):** Present value of a perpetuity whose periodic payment is $Rs.R$, interest being $r\%$ per period or $\frac{r}{100} = i$ per period per rupee is given by $\frac{R}{i}$.
Reason (R): Sum of an infinite GP whose first term is a and common ratio is r is given by $\frac{a}{1-r}$.

SECTION-B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. Find all the positive integers less than 30 forming the equivalence class of 5 for modulo 7.
OR

Cost of two types of pulses is Rs.55 per kg and Rs.90 per kg. If both the pulses are mixed together in the ratio 2:3. What should be the price of the mixed variety of pulses per kg?

22. How long will it take to row 20 km upstream if one can row 10 km in 10 minutes in still water and the same distance in 8 minutes with the stream?

23. A cistern can be filled by two pipes A and B in 12 minutes and 15 minutes respectively. Another tap C can empty the full tank in 20 minutes. If the tap C is opened 5 minutes after the pipes A and B are opened, find when the cistern becomes full?

24. If $A = \begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix}$, find $f(A)$, where $f(x) = x^2 - 2x + 3$.

25. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of 6 workers 4 or more will catch the disease?
OR
The lifetime of an item produced by a machine has a normal distribution with mean 12 months and standard deviation of 2 months. Find the probability of an item produced by this machine will last
i) less than 7 months ii) between 7 and 14 months.
(Given $P\left(Z < \frac{5}{2}\right) = 0.9938$ & $P(Z < 1) = 0.8413$)

Section-C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Find the remainder when $987 + 876 + 765 + 654 + 543 + 432 + 321 + 210$ is divided by 6.

27. A 95% confidence interval for a population mean was reported to be 152 to 160. If $\sigma = 15$, what sample size was used in this study? (Given $Z_{0.05} = 1.645, Z_{0.025} = 1.96$)

    OR

    A fertiliser company packs the bags labelled 50 kg and claims that the mean mass of bags is 50 kg with a standard deviation of 1 kg. An inspector points out doubt on its weight and tests 60 bags. As a result, he finds that mean mass is 49.6 kg. Is the inspector right in his suspicions? (Given $\alpha = 0.05$)

28. A box contains 200 tickets, each bearing one of the numbers from 1 to 200. 20 tickets are drawn successively with replacement from the box. Find the probability that at most 4 tickets bear numbers divisible by 20. (Given $\frac{1}{e} = 0.368$)

29. A company manufactures two types of cardigans: type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigan. The company can make at most 300 cardigans and spend at most & 72000 a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than 200. The company makes a profit of ₹100 for each cardigan of type A and ₹50 for every cardigan of type B. Formulate only this problem as a linear programming problem to maximise the profit of the company.

30. If X is normally distributed with mean 6 and standard deviation 5, find

    i) $P(0 \leq X \leq 8)$ ii) $P(|X - 6| < 10)$.

    (Given $P(0 < Z < 1.2) = 0.3849, P(0 < Z < 0.4) = 0.1554, P(0 < Z < 2) = 0.4772$)

    OR

    According to an educational board survey it was observed that class 12 students apply at least one to four weeks ahead of college application deadline and the probability of student to get admission in the college $P(X = x)$ is given as follows:

$$P(X = x) = \begin{cases} \frac{kx}{6}, when\ x = 0,1\ or\ 2 \\ \frac{(1-k)x}{6}, when\ x = 3 \\ \frac{kx}{2}, when\ x = 4 \\ 0, when\ x > 4 \end{cases}$$ where k is a real number.

    Based on the above information, answer the following questions:

    i) Find the value of k.

    ii) Calculate the mathematical expectation of number of weeks taken by a student to apply ahead of a college's application deadline.

31. Mr. Kumar has invested Rs.20000 in year 2014 for 5 years. If CAGR for that investment turned out to be 11.84%. What will be the end balance? (Given $(1.1184)^5 = 1.75$)

Section-D

(This section comprises of long answer type questions (LA) of 5 marks each.)

32. A machine costing Rs.200000 has effective life of 7 years and its scrap value is Rs.30000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost Rs.300000 after 7 years. (Given $(1.05)^7 = 1.407$)

33. An insect is crawling along the path $y = ax^2 + bx + c$, where $a \in R - \{0\}$, $b, c \in R$. It passes through the points $(2,18), (3,41)$ and $(1,5)$. Using Cramer's Rule, find the values of a, b and c.

OR

Find A^{-1}, where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$. Hence, solve the following system of equations by matrix method: $4x + 2y + 3z = 2, x + y + z = 1, 3x + y - 2z = 5$.

34. A tyre manufacturer estimates that (thousand) radial tyres will be purchased i.e. demanded by whole sales when price is $p = D(x) = 90 - \frac{x^2}{10}$ thousand rupees per tyre and the same number of tyres will be supplied when the price is $p = S(x) = \frac{1}{5}x^2 + x + 5$ thousand rupee per tyre. Find the equilibrium price and the quantity supplied and demanded at that price. Also, find the consumer surplus at the equilibrium price

35. Fit a straight-line trend by the method of least squares and find the trend value for the year 2008 for the following data:

Year	2001	2002	2003	2004	2005	2006	2007
Production (in lakh tons)	30	35	36	32	37	40	36

OR

The following table shows the annual rainfall (in mm) recorded for Churaumi, Meghalaya:

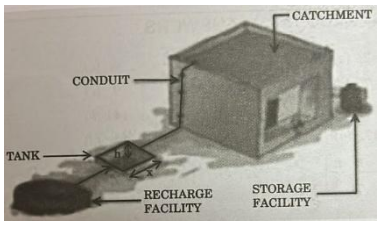
Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Rainfall (mm)	1.2	1.9	2	1.4	2.1	1.3	1.8	1.1	1.3

Determine the trend of rainfall by 3-year moving average and show these moving averages on a graph paper.

SECTION- E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each)

36. In order to set up rainwater harvesting system a tank to collect rainwater is to be dug. The tank should have a square base and a capacity of 250 m^3 the cost of land is Rs.5000 but a square meter and cost of digging increases with depth and for the whole tank it is Rs.40,000 m^2, where h is the depth of the tank in meter. x is the side of the square base of the tank in meters.



On the basis of above information, answer the following questions:

i) Find the total cost C of digging the tank in terms of x.

ii) Find $\frac{dC}{dx}$.

iii) Find the value of x for which cost C is minimum.

OR

Check whether the cost function $C(x)$ expressed in terms of x is increasing or not, where $x > 0$.

37. A machine purchased for ₹2,00,000 has effective life of 5 years and its scrap value is Rs.25,000. It is assumed that the cost of the machine will increase by 3% every year. The prevailing interest rate is 5% per annum.

(Given $(1.03)^5 = 1.1593, (1.03)^{10} = 1.3439, (1.05)^5 = 1.2763$)

Based on the above information answer the following questions:

i) Find the annual depreciation using linear method.

ii) Find the cost of the machine after 5 years.

iii) Find the amount of money that should be deposited at the end of the year into the sinking fund to purchase the new machine.

OR

If the rate of interest is 6% per annum compounded half yearly, then find the amount to be deposited at the end of every 6 months, to be able to make the purchase.

38. A manufacturer has three machines M_1, M_2 and M_3 installed in his factory. Machines M_1 and M_2 are capable of being operated for at-most 12 hours, whereas machine M_3 must be operated for at least 5 hours a day. The manufacturer produces only two items, each requiring the use of these three machines for producing 1 unit of A or B.

Items	Number of hours required on the machines		
	M_1	M_2	M_3
A	1	2	1
B	2	1	5/4

He makes a profit of ₹60 on item A and ₹40 on item B.

Based on the above information, answer the following questions:

i) Formulate the objective function and constraints of the above Linear programming problem.

ii) How many of each item he should produce to have maximum profit?